(S.03)

Find 2 fold eqn of two lines inclined by an Angle B on the line $x+y=4$, and passes through (2,2) if $B = 30°, 60°, 15°$

Sol$_2$

Translation to (2,2)

$x = X + 2$, $y = Y + 2$

@ $x + y = 4$

$X + 2 + Y + 2 = 4$

$B_1: X + Y = 0$

$B_2: X - Y = 0$ $\rightarrow$ $\boxed{X^2 - Y^2 = 0}$

$$\frac{X^2 - y^2}{a - b} = \frac{Xy}{h}$$

$a = b$

$h = \pm a \operatorname{Sec}(2B)$

$a X^2 + b y^2 + 2h Xy = 0$

$a X^2 + a y^2 \pm 2a \operatorname{Sec}(2B) Xy = 0$

$(x-2)^2 + (y-2)^2 \pm 2 \operatorname{Sec}(2B) (X-2)(Y-2) = 0$

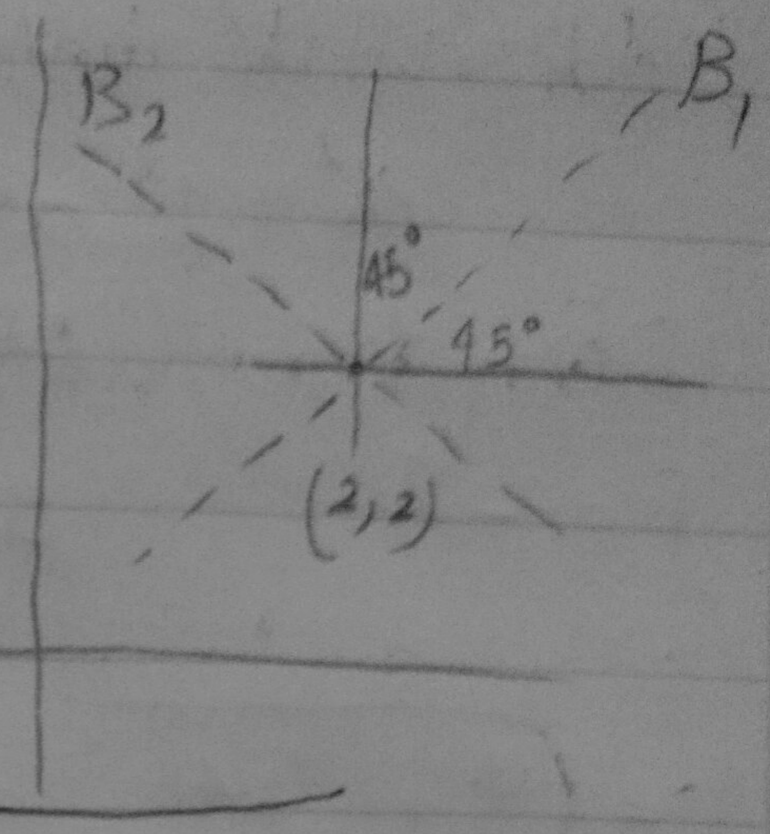
$X^2 + y^2 \pm 2 \operatorname{Sec}(2B) (Xy - 2X - 2y) - 4(x+y) + 8 = 0$

① $B = 30°$, $2B = 60°$ $\operatorname{Sec} 60° = 2$

⊕ $x^2 + y^2 + 4xy - 12x - 12y + 24 = 0$ ⊖ $x^2 + y^2 - 4xy + 4x + 4y - 8 = 0$

$B = 45°$

الحل من الرسم



$L_1 : X = 0 \Rightarrow X - 2 = 0$

$L_2 : Y = 0 \Rightarrow Y - 2 = 0$

$(X-2)(Y-2) = 0$

Circle :.

* $X^2 + Y^2 + 2fX + 2gY + C = 0$

[1] C.o.f X^2 = C.o.f Y^2

[2] XY term $= 0$

[3] Center : $(-f, -g)$

$$r = \sqrt{f^2 + g^2 - C}$$

* another form :.

$$(X+f)^2 + (Y+g)^2 = r^2$$

Subject:

Date:

circle

EX: write down the center & radius of this eqn

$$x^2 + y^2 - 4x - 8y - 41 = 0$$

$2f = -4 \quad \therefore \quad 2g = -8$

$f = -2 \qquad g = -4$

$r = \sqrt{4 + 16 + 41} = \sqrt{61}$

center: $(2, 4)$

Deduce eqn of circle of which its center $(4, -6)$

$r = 7$ ∴

Sol

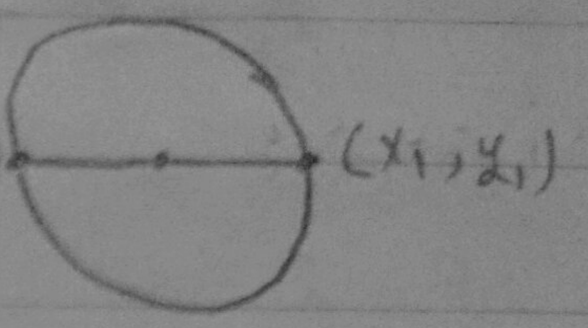
$f = -4 \quad , \quad g = 6$

$$(x - 4)^2 + (y + 6)^2 = 49$$

$$x^2 + y^2 + 12y - 8x + 3 = 0$$

eqn of circle using Two ends of Diameter :-

$$(X - X_1)(X - X_2) + (Y - Y_1)(Y - Y_2) = 0$$



Ex:- $(1, -2)$, $(4, -3)$ $\Rightarrow$ Tw. ends of Diameter

Find eqn of circle

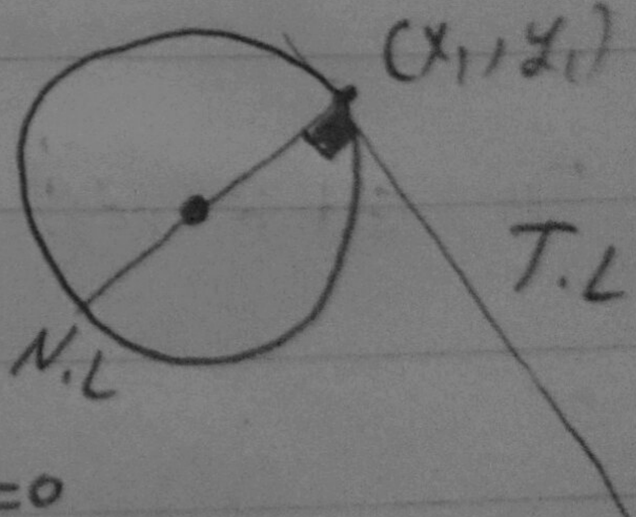
$$(X - 1)(X - 4) + (Y + 2)(Y + 3) = 0$$

$$X^2 + Y^2 - 5X + 5Y + 10 = 0$$

Tangent Line and normal Line eqns :-

T.L :-

$$X^2 + Y^2 + 2PX + 2gY + C = 0$$



$$T.L : XX_1 + YY_1 + F(X + X_1) + g(Y + Y_1) + C = 0$$

$$N.L \quad \frac{Y - Y_1}{X - X_1} = \frac{Y_1 + g}{X_1 + F}$$

Subject. Date.

EX:- Find N.L , T.L For the circle

$$x^2 + y^2 + 2x - 3y - 3 = 0 \quad \text{at} \quad (1, 3)$$

Sol:-

$2f = 2 \qquad 2g = -3$

$f = 1 \qquad g = -\frac{3}{2} \qquad c = -3$

$$x + 3y + 1(x+1) + \left(\frac{-3}{2}\right)(y+3) - 3 = 0$$

$$T.L: 4x + 3y - 13 = 0$$

$$N.L: \frac{y-3}{x-1} = \frac{3 + \frac{-3}{2}}{1+1} = \frac{4}{3}$$

$$3x - 4y + 9 = 0$$

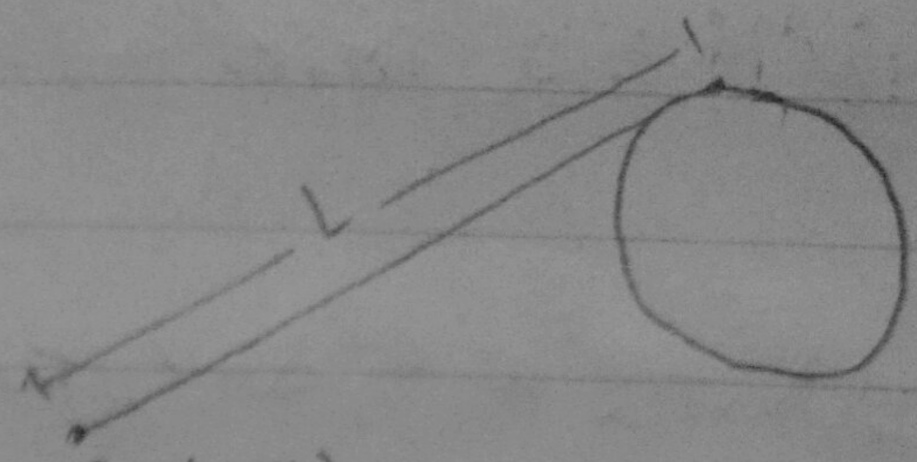
Subject

Date

Tangential Length :-

$$L^2 = x_1^2 + y_1^2 + 2fx_1 + 2gy_1 + c$$

$$L = \sqrt{L^2}$$



① $L^2 = 0$, B on the circle

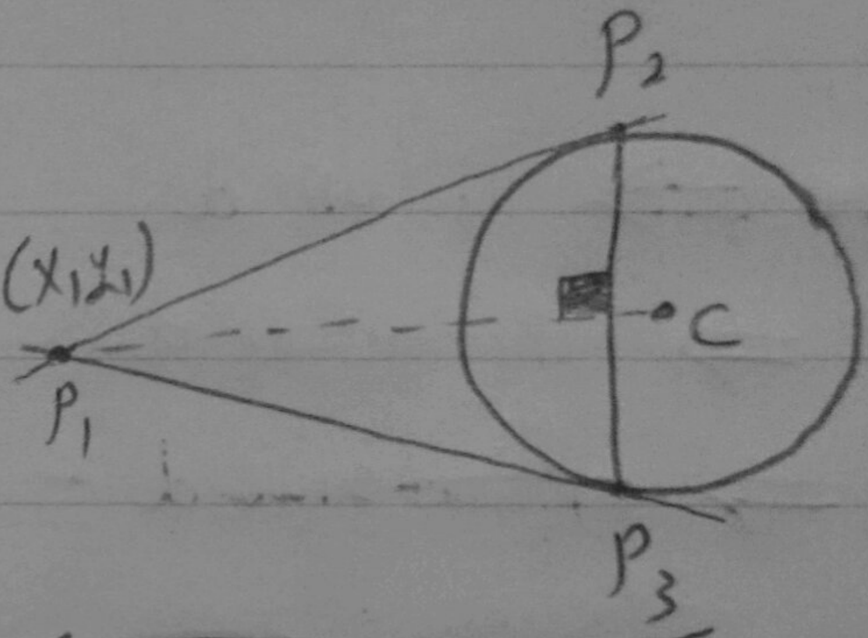
② $L^2 > 0$, B out of //

③ $L^2 < 0$; B inside //

Tangential chord :-

T.C.

$$xx_1 + yy_1 + f(x+x_1) + g(y+y_1) + c = 0$$



Find T.L for the circle

$$x^2 + y^2 + 6x - 4y - 3 = 0 \quad \text{at } (0,6)$$

*check (0,6) position for the circle :-

$$L^2 = 0 + 36 + 0 - 4\times6 - 3 = 9 > 0 \quad \text{out of the circle}$$

$$T.C: \; 3x + 4y - 15 = 0 \Rightarrow y = \frac{15-3x}{4}$$

$$x^2 + \left(\frac{15-3x}{4}\right)^2 + 6x - 4\left(\frac{15-3x}{4}\right) - 3 = 0 \Rightarrow 5x^2 + 54x + 63 = 0$$

$$x_1 = \frac{21}{25} \quad \text{ó} \quad x_2 = -3$$

$$y_1 = \frac{78}{25} \quad \text{ó} \quad y_1 = 6$$

$$T.L_1 : y = 6 \qquad \text{at } (-3,6)$$

$$T.L_2 : 96x + 28y - 168 = 0 \quad \text{at } \left(\frac{21}{25}, \frac{78}{25}\right)$$